Divide and Conquer Neural Networks for Two-Dimensional and Three-Dimensional Electrical Impedance Tomography

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Electrical Impedance Tomography (EIT) is a relatively modern approach for imaging applications, and is a promising technology for biomedical applications. By applying an electrical current to a living tissue and measuring the electrical potential at different points of the boundary of the tissue, it is possible to solve the inverse problem and allow us to generate a map of the conductivities of the tissue. However, since this is an ill-posed problem, solving a non-linear EIT problem is usually done by linear inverse solvers in order to regularize and reduce the ill-posedness of the problem. Recently, use of Artificial Neural Network (ANN) to solve an EIT inverse problem has been proposed. ANNs are capable of non-linear approximations, and can give a nonlinear conductivity distribution. Although several works have been published on 2-dimensional FE models, very little work has considered three-dimensional problems. In this paper, a solution based on Divide and Conquer and ANNs is used to solve the non-linear inverse problem without any linearization. The solution presented here aims to reduce the difficulty of training a large ANN, which is commonly required to solve a 3D-EIT problem with Artificial Intelligence (AI) algorithms.

Index Terms— **Artificial Neural Networks, Complexity, Electrical Impedance Tomography, Inverse Problems**

I. Introduction

In electrical impedance tomography (EIT), the goal is to obtain the distribution of the internal electrical conductivities of living tissues by measuring the electrical potential at the boundary of the body [1]. In biomedical applications, this technique allows both inexpensive and radiation free tomography [2], which is a significant improvement on biomedical scanners currently used in hospitals. In EIT, internal conductivities can be obtained from the voltages measured at the boundary of the Finite Element (FE) model by solving a nonlinear inverse problem. This nonlinear inverse problem is very challenging since the commonly used inverse solvers tend to linearize the problem, inducing smoothness and artifacts in the solution [3].

Some researchers proposed the use of Artificial Neural Networks (ANNs) algorithms to solve the inverse problem. ANNs are a series of algorithms, which are capable of learning how to solve any kind of nonlinear problem. Several papers studied ANN as a replacement of the linear inverse solver for Two-Dimensional (2D) problems [4], but the available literature on solving three-dimensional (3D) problems is still limited. This limitation can be due to the difficulties of efficiently training an ANN which is capable of solving the EIT problem in 3D with greater accuracy.

This paper introduces an original method to divide the 3D EIT reconstruction problem into manageable subtasks to allow the reconstruction of 3D EIT images more efficiently. The FE model is divided into several parts and each part of the FE model is solved by a different ANN. This approach limits the number of outputs of the ANN, and consequently reduces the number of nodes, weights, and biases, which makes the training process much faster and gives a greater convergence.

II. Theory

EIT images can be simulated in two steps, commonly known as the forward problem and the inverse problem. In the forward problem, given a conductivity distribution within a FE model, the goal is to determine the corresponding voltages at the boundary. This step can be solved linearly by using a linear solver [1]. On the other hand, a specific set of measurements may correspond to several different conductivity distributions, making the inverse problem severely ill-posed [5]. A common practice is to assume a linear conductivity distribution. This assumption reduces the ill-posedness aspect of the EIT inverse problem, and allows one to utilize a linear inverse solver to solve the problem.

Although this assumption gives good results, it cannot reconstruct an exact result when the conductivities are nonlinearly distributed. The problem is that, in real applications, the electrical current passes through several different tissues and each of these tissues present rough boundaries, at which the conductivity change is nonlinear. Different algorithms, based on supervised learning, have been proposed to replace the linear inverse solver. These algorithms are capable of finding a solution to a nonlinear problem in a very short time. ANNs are capable of very good approximations and can be used to solve the EIT inverse problem in 2D [6]. However, the complexity of a 3D FE model, in terms of available measurements and nodes, will automatically lead to an exponential growth of the ANN in terms of input and output neurons. It has been shown that training such a large ANN usually takes more time, and requires a greater number of training data. After training the ANN, the testing phase shows that, the larger the dimension of the ANN, the lower the convergence, and the higher the risk of falling into local minima.

Previous work has shown that it is possible to reduce the complexity of the training process by dividing the problem into several ANNs using Divide and Conquer (D&C) [7]. Here, the FE model is divided into several 'sub meshes', and each of these sub-meshes is solved with a specific ANN. By doing this, it is possible to significantly reduce the number of weights and biases in the network, allowing a more efficient training, in terms of accuracy and computation requirement.

III. Result

In this example, the FE model was divided into 64 'sub meshes', each of them containing a variable number of output neurons, between 400 and 500. The whole mesh contained 27333 elements, and 4 layers of 8 electrodes, allowing up to 928 different measurements. Each ANN was given the set of 928 measured voltages as input data, and gave the conductivity distribution of a specific sub-mesh at its output. The FE model was divided on both azimuthal and vertical axis. In both azimuthal and vertical directions, the FE model was divided into 8 different sections. It was possible subsequently to combine the output of each ANN to obtain the reconstructed image.

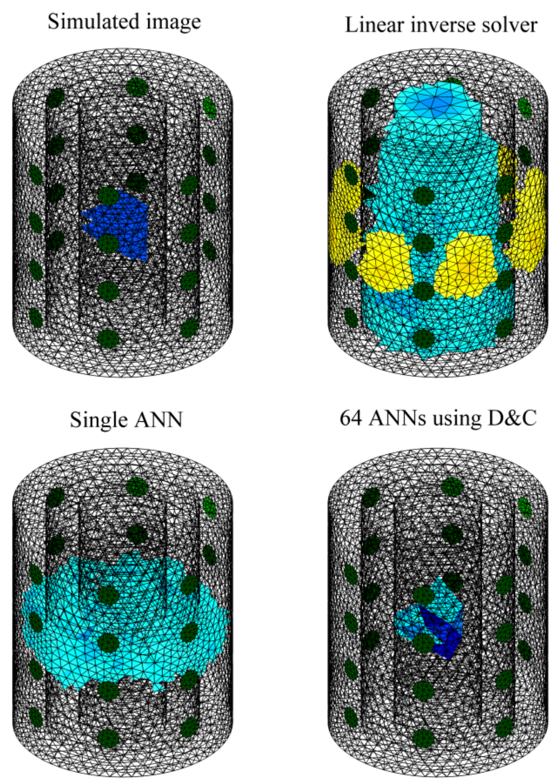

Fig. 1 Comparison of 3D-EIT images obtained with a linear solver, one ANN and the proposed method with 64 ANNs

Fig. 1 shows an EIT image and its corresponding reconstructions obtained with linear inverse solver (Gauss Newton One Step), one ANN, and 64 ANNs with the proposed D&C ANN algorithm. It can be seen that the training algorithm had difficulties in training the ANN with 27333 outputs. The proposed method gives a higher accuracy than the linear inverse solver or the single ANN.

Table 1 RMS errors obtained for 3D-EIT reconstructions using the linear inverse solver, one ANN and the proposed method (64 ANNs with D&C)

Linear inverse solver	one ANN	Proposed D&C method
31.48 %	15.10 %	5.83%

After reconstruction and normalization, a corresponding Root Mean Square (RMS) error is calculated based on the conductivity of each element in the FE model. The resulting errors are reported in Table 1. It can be seen that the proposed method significantly reduces the RMS error obtained on the image reconstruction. Basically, this is a consequence of the non-linear behavior of the ANN. The images obtained with the proposed algorithm show that the artifacts resulting from linearization are significantly reduced when the inverse problem is solved with ANN.

Another interesting advantage of using ANN to solve the EIT inverse problem is their strong ability to deal with noisy input data when including the noise model to the training samples [8]. An Additive White Gaussian Noise (AWGN) and the presence of a hardware band pass filter have been considered in the training phase, and the measured voltages adjusted accordingly. By considering noise that is present in the real systems, the proposed method is able to generate a better reconstruction under noisy conditions.

IV. Conclusion

To conclude, the proposed D&C method in which the 3D problem is divided into several 'sub-problems', in conjunction with ANN algorithm, allows the 3D-EIT inverse problem to be solved by a fully nonlinear algorithm with a significantly higher degree of accuracy than a simple ANN algorithm and traditional inverse solver. The D&C method reduces the complexity of the ANN algorithm, and increases its global convergence.

The proposed method also shows good resistance to noise. ANNs trained with noisy data are capable of solving the inverse problem with a noisy input. This ability is very important for real-world measurements, where an extremely low-current is injected, and where the noise may become an important source of errors in the reconstruction process.

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